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**478. Proposed by CLIFFORD N. MILLS, Brookings, S. Dak.**

Solve the equations,  $l/x + y/m + z/n = 1$ ,  $x/l + m/y + z/n = 1$ ,  $x/l + y/m + n/z = 1$ .

SOLUTION BY HENRY D. THOMPSON, Princeton N. J.

Take  $X, Y, Z$  as symbols for  $x/l, y/m, z/n$ , respectively. Then the equations to be solved are:

$$(1) 1/X + Y + Z = 1, \quad (2) X + 1/Y + Z = 1, \quad (3) X + Y + 1/Z = 1.$$

The difference of (2) and (1) gives (4)  $(X - Y) + (X - Y)/XY = 0$ , one solution of which is (5)  $X = Y$ . This substituted in (1), (2), (3) leaves to be solved (6)  $1/X + X + Z = 1$ , (7)  $2X + 1/Z = 1$ , and (7) gives  $Z = 1/(1 - 2X)$ .

This in (6) gives the cubic

$$1/X + 1/(1 - 2X) = 1 - X, \quad \text{or} \quad (1 - X)/X(1 - 2X) = (1 - X),$$

the solutions of which are  $X = 1$ , and  $X = 1/4 \pm \sqrt{-7}/4$ . These with (5) and (7) give for  $[x, y, z]$  the three solutions:

$$(I, II) [(1 \pm \sqrt{-7})l/4, (1 \pm \sqrt{-7})m/4, (1 \pm \sqrt{-7})n/4], \quad (III) [l, m, -n].$$

The other solution of (4) viz.: (8)  $1 + 1/XY = 0$  in (1) or (2) gives  $Z = 1$ , whence (3) becomes  $X + Y = 0$ , which with (8) gives  $X^2 - 1 = 0$ , and  $X = 1$  with  $Y = -1$ , or  $X = -1$  with  $Y = 1$ . Two other solutions are:

$$(IV) [l, -m, n], \quad (V) [-l, m, n].$$

To find the infinite solutions, set  $X/W, Y/W, Z/W$ , respectively, for  $X, Y, Z$  in (1), (2), (3) and these equations rid of denominators, and with  $W = 0$  are:

$$(9) X(Y + Z) = 0, \quad (10) Y(Z + X) = 0, \quad (11) Z(X + Y) = 0.$$

The equation (9) is satisfied when  $X = 0$ , which set in (10) or (11) gives either  $Y = 0$  or  $Z = 0$ . The other solution of (9) viz.: (12)  $Y = -Z$  set in (10) and (11) gives  $Y(X - Y) = 0$  and  $-Y(X + Y) = 0$ ; whence  $Y = 0$ , and from (12),  $Z = 0$ . Thus the infinite solutions (VI), (VII), (VIII) are the points at infinity on the axes when  $x, y, z$  are taken as coördinates in a rectilinear system.

Note that each of the three hyperbolic cylinders (1), (2), (3) has for its asymptotic planes a coördinate plane and a plane parallel to the like named axis.

Also solved by B. J. DINING, GERTRUDE I. MCCAIN, J. L. RILEY, E. F. CANADY, S. W. REAVES, J. Q. McNATT, G. W. HARTWELL, R. A. JOHNSON, C. H. WORTHINGTON, PAUL CAPRON, HORACE OLSON, and G. Y. SOSNOW.

**479. Proposed by S. A. COREY, Albia, Iowa.**

Prove or disprove

$$\left\{ \begin{vmatrix} x & -v & -z \\ -y & -z & v \\ -z & y & -x \end{vmatrix}^2 + \begin{vmatrix} y & -v & -z \\ x & -z & v \\ v & y & x \end{vmatrix}^2 + \begin{vmatrix} x & y & -z \\ -y & x & v \\ -z & v & -x \end{vmatrix}^2 + \begin{vmatrix} x & -v & y \\ -y & -z & x \\ -z & y & v \end{vmatrix}^2 \right\} \\ \div \begin{vmatrix} x & -y & -z & v \\ y & x & -v & -z \\ z & v & x & y \\ v & -z & y & -x \end{vmatrix}^2 = (x^2 + y^2 + z^2 + v^2)^{-1}.$$

SOLUTION BY HENRY D. THOMPSON, Princeton, N. J.

Let  $Q$  be a symbol for  $(x^2 + y^2 + z^2 + v^2)$ , and represent the determinants in the order in which they appear in the equation by  $Z, V, X, Y, H$ , so that the equation is

$$\{Z^2 + V^2 + X^2 + Y^2\} \div H^2 = Q^{-1}.$$

Application of the rule for the multiplication of determinants gives:

$$Z^2 = \begin{vmatrix} Q - y^2 & -yx & -yv \\ -xy & Q - x^2 & -xv \\ -vy & -vx & Q - v^2 \end{vmatrix}.$$

Dividing the first, second, and third columns by  $y, x, v$ , respectively, and multiplying the first, second, and third rows, respectively, by the same symbols gives:

$$Z^2 = \begin{vmatrix} Q - y^2 & -y^2 & -y^2 \\ -x^2 & Q - x^2 & -x^2 \\ -v^2 & -v^2 & Q - v^2 \end{vmatrix},$$

which easily reduces to  $Z^2 = Q^2(Q - v^2 - y^2 - x^2) = Q^2z^2$ .

In exactly the same way,  $V^2 = Q^2v^2$ ,  $X^2 = Q^2x^2$ ,  $Y^2 = Q^2y^2$ . Whence

$$\{Z^2 + V^2 + X^2 + Y^2\} = Q^2(z^2 + v^2 + x^2 + y^2) = Q^3.$$

Direct application of the rule for the multiplication of determinants gives for  $H^2$  a determinant wherein each element in the leading diagonal is  $Q$ , and all the other elements are zero; whence  $H^2 = Q^4$ . These values set in the equation prove it to be correct.

Also solved by J. L. RILEY, E. H. WORTHINGTON, A. M. HARDING, J. B. ROSENBAUGH, and the PROPOSER.

#### GEOMETRY.

##### 511. Proposed by FRANK V. MORLEY, Student, Haverford, College.

Let  $a_i$  ( $i = 1, 2, 3, 4$ ) be four points on a circle, and let the incenter of the triangle formed by omitting  $a_i$  be  $c_i$ ; prove that the four points  $c_i$  form a rectangle.

##### SOLUTION BY THE PROPOSER.

Let there be four circles with centers,  $m_{12}, m_{23}, m_{34}, m_{41}$ , on the given circle, and let each circle intersect the next in two points, of which one set,  $a_2, a_3, a_4, a_1$ , lie on the given circle, and one set,  $c_4, c_1, c_2, c_3$ , lie inside, as shown in Fig. 1.

Then  $\sphericalangle c_4a_1a_2 = \frac{1}{2}\sphericalangle c_4m_{12}a_2$ , and  $\sphericalangle m_{23}a_1a_2 = \frac{1}{2}\sphericalangle a_3m_{12}a_2$  (1).

$\therefore \sphericalangle c_4c_3a_2 = \sphericalangle c_4a_1a_2 = \sphericalangle m_{23}a_1a_2 = \sphericalangle m_{23}m_{41}a_2$ , and  $c_3c_4 \parallel m_{41}m_{23}$ .

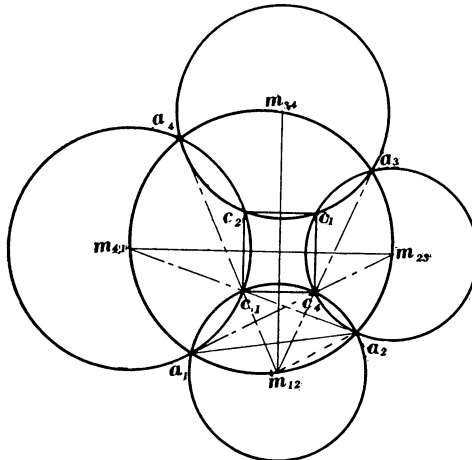


FIG. 1.

By equality of arcs,  $m_{41}m_{23} \perp m_{12}m_{34}$ , and hence  $c_3c_4 \perp m_{12}m_{34}$ .

Since  $\sphericalangle a_4m_{12}a_3 = \sphericalangle c_3m_{12}c_4$ , and is bisected by  $m_{12}m_{34}$ ,  $c_3, c_4$  are reflections through  $m_{12}m_{34}$ , and likewise  $c_1, c_2$ ; similarly  $c_1, c_4$ , and  $c_2, c_3$ , are reflections through  $m_{41}m_{23}$ .

$\therefore c_1, c_2, c_3, c_4$ , form a rectangle.